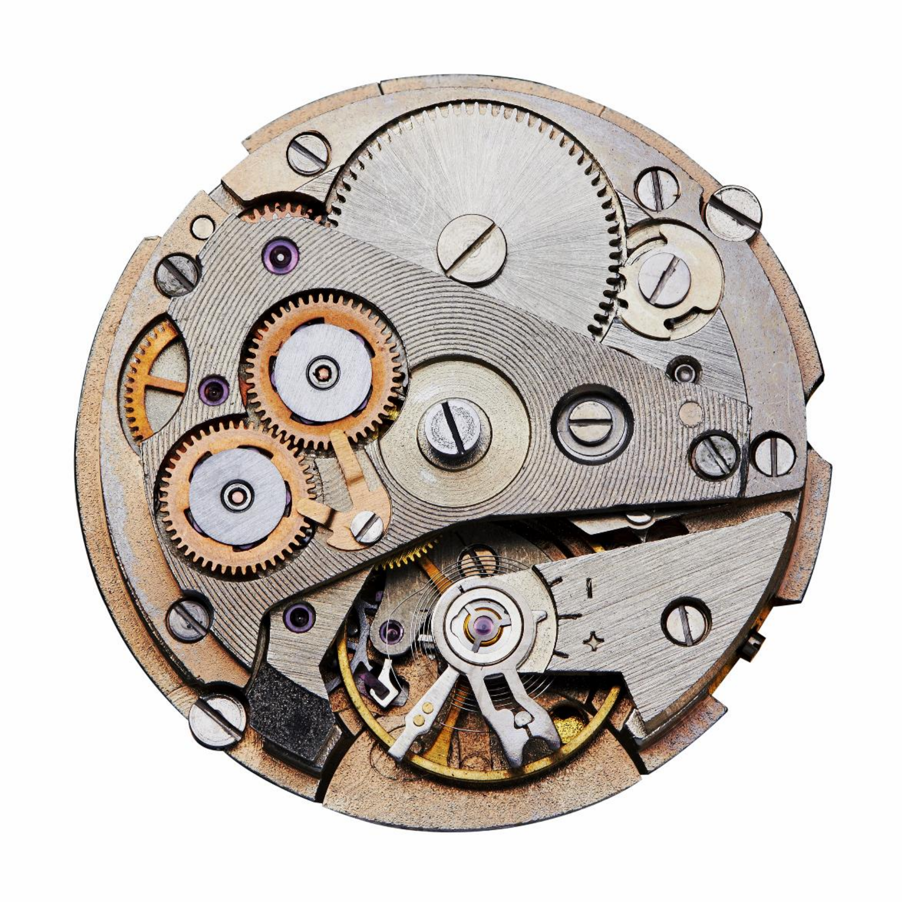

Simultaneous calibration of an interest model to multiple valuation dates

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What is calibration

…looking at physics

Newtonian Physics – gravity:

Speed of object in free fall after time t: $v(t) = gt$

→ Distance covered after time t: $s(t) = {}^{1}/_{2}\, gt^2$

Calibration: Estimate the global gravity constant: $g \approx 10\ {}^{m}/_{s^2}$

Without Newton

Normal model $s(t) = \alpha t$

Square root model $s(t) = \beta t^{3/2}$

Lognormal model $s(t) = \gamma \exp(t)$

Calibration

t	Observation of s	α	β	γ
1	5	5.0	5.0	1.8
2	20	10.0	7.1	2.7
3	45	15.0	8.7	2.2
4	80	20.0	10.0	1.5

(Idea from Paul Wilmott, „Bankers can't avoid risk by hiding it")

Context

Valuation of Guarantees in Life Insurance

Object of interest:

- Typical product: annuity with some sort of guarantee and profit sharing
- A very long-running product: expected to be on the books for 60 years or so

Task to perform:

- Need to value liability for various reasons (Pricing, risk management, regulatory, accounting).
- Also need to be able to accurately contrast different products:
  - Product design choices
  - Fair differentiation of profit sharing with consideration of guarantee cost
- Value given hypothetical market changes (i.e. yield curve stresses)

In a (very inaccurate) high-level perspective, valuing financial instruments with maturity similar to the market instruments is model-based *interpolation*, while we are effectively *extrapolating* prices in term and in market conditions.

→ We would like our models to be stable enough to extrapolate reasonably.

At the same time, fitting to market data as at the valuation date is mandatory.

Baseline: one factor Hull-White-Modell

Reference group: German life insurance companies calculating Solvency II

Most used model: The GDV publishes a 1-Factor-Hull-White model for use of its members (market share in terms of number of insurance companies 50%-75%). The DAV provides example calibrations (and calibration methodology).

In their setup

$$dr_t = \alpha(\theta(t) - r_t)dt + \sigma dW_t$$

$\theta(t)$ is given by the risk-free yield curve (either swap or EIOPA) and no-arbitrage.

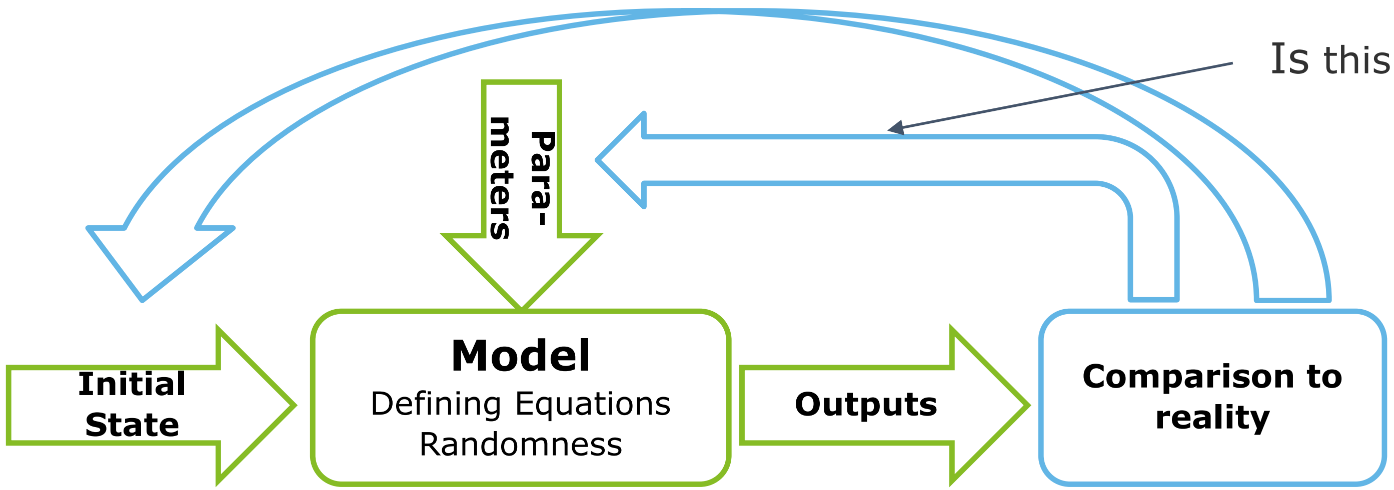
DAV proposes

- Setting $\alpha = 0.1$ as this is „consensus among practitioners“
- Calibrate σ to replicate the price of a 10-into-10 swaption.

GDV = Association of German Insurers
DAV = German Actuarial Association
(source „Zwischenbericht zur Kalibrierung und Validierung spezieller ESG unter Solvency II“, November 2015)

What are we doing when we calibrate a model?

Is this a good idea?



Hull White:

$$dr_t = \alpha(\theta(t) - r_t)dt + \sigma dW_t$$

Fixed parameter,
„characteristic of the market“

Parameter
Calibrated to current **state**

Looks like a parameter, but can be made
initial state in a Forward-Rate reformulation.

The “obvious initial state” r_0 is not interesting when using HW ⇔ Vasicek + Shift.

Possible modes of operation

Calibration of the model for each valuation date (most common approach)

Use most parameters to fit to market data

Usually Black-Box-Optimization

- Relatively straightforward
- Conflicting targets need to be prioritised in terms of weighted penalties
- Often, users have limited intuition regarding the various parameters
- Sometimes unexpected effects (e.g. degenerated parameters, implausible sensitivities)

It can be instructive to manually optimize parameters to gain intuition regarding the effect of each parameter, but it is boring in the long run

Example seen in practice

Recalibrate everything

CIR++ Model:

$$dx_t^i = \alpha_i\left(\theta_i - x_t^i\right)dt + \sigma_i\sqrt{x_t^i}dW_t$$

$$r_t = \sum_i x_t^i + \varphi(t)$$

Calibration:

- Fit all of α_i, θ_i, σ_i and φ (piecewise) to atm swaption surface.
- Recalibrate in stresses to match Black swaption volatilities.

Calibration can be unstable.

Possible modes of operation

Global calibration
(exotic in practice, theoretically appealing)

Only use model state to fit to market data

- somewhat exotic, but actually occurring in practice with „classical” models
- Conceptually pleasing (parameters as „market characteristics“ derived from observations)
- What if states are impossible to reach?
  e.g. interest falls below lower bound

Example from academic literature

D. Filipović, M. Larsson, and A.B. Trolle. "Linear-Rational Term Structure Models." *The Journal of Finance* 72.2 (2017): 655-704.

Focus slightly different (in their own words):

- ensures nonnegative interest rates
- easily accommodates unspanned factors affecting volatility and risk premiums, and
- admits semi-analytical solutions to swaptions

(There probably are more that I am unaware of.)

Globally calibrating Hull-White

Error in normal swaption volatilities with fixed σ

Procedure:

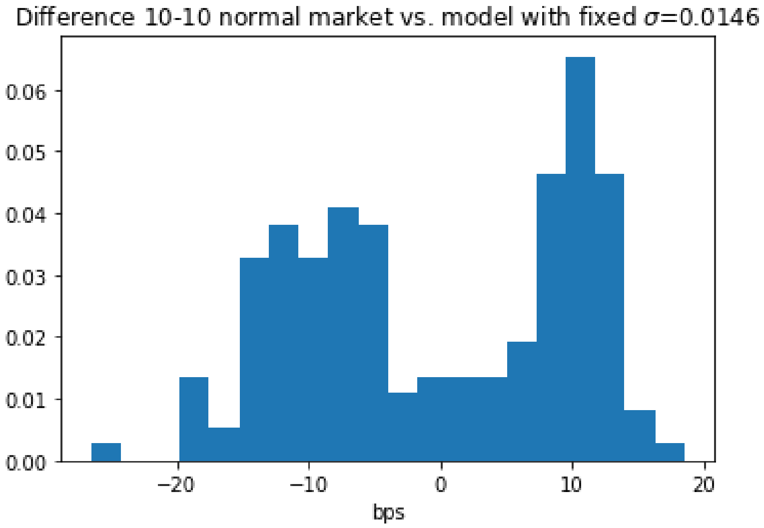
- Determine σ minimizing total squared difference for end of months 2003-2017
- Evaluate difference market vs. model normal volatility for a 10-10 at the money swaption for each date.

Result:

- Little mass close to 0 error (= good fit), most around +/- 10bps.
- Model volatility σ between 2003-2008 and 2009-2017 levels.

→ No market consistence.

Empirical density of difference:



Possible modes of operation

Pragmatic approach – calibrate model globally and „fine-tune" to market data

Divide model inputs into three categories

- Global parameters representing market characteristics
- True states per the calibration date
- Parameters used like state to fit market values at the calibration state
  → Ideally, the variability of the latter should be of limited impact to the valuation.

Benefits:

- Stable calibration, ideally no recalibration necessary for sensitivities (e.g. Solvency stresses, but also planning)
- More reliable results when stress scenarios actually happen

So we need

- a model that facilitates the above
- a methodology to derive global parameters

Pragmatic Hull-White

Calibration of σ to the 10-10 swaption for a number of valuation dates

Use baseline method. Procedure:

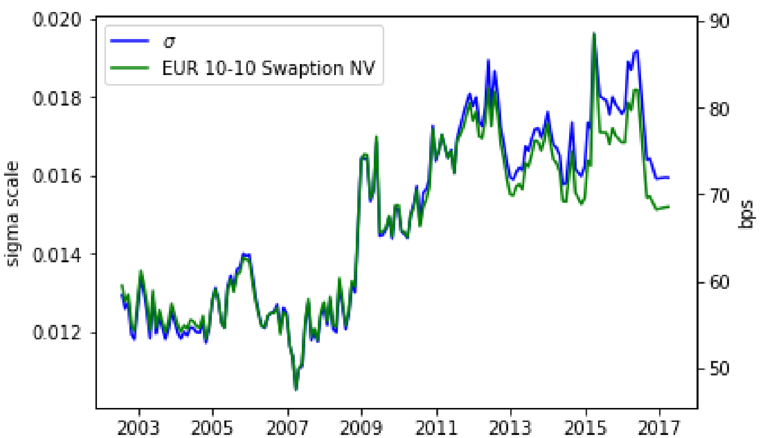
- Fix α globally.
- For each date (end of month for which complete data was available), compute σ matching quoted normal volatility.

→ Significant variability, even when considering 2003-2008 separately from 2009-2017.

So we would like some time-varying volatility, but perhaps not “all of it”.

Also, we might want some more detailed control on interest rate range.

→ Probably easier to achieve in forward setting.



Possible modes of operation

Pragmatic approach – a candidate model

- Forward-rate model (simpler framework)
- Volatility dependence on interest rate level inspired by Deguillaume, Rebonato, Pogudin (see graphic)
  → "Morally" closer to having a real-world (physical measure) counterpart than many "purely risk-neutral" LMM-type models
  → Much more reasonable behaviour in long projectons and sensitivities (without recalibrating) than many other models.

- Term-dependent lower bound to can be prescribed a priori (this lies below zero)

- We will not get an analytical swaption price formula – that's OK

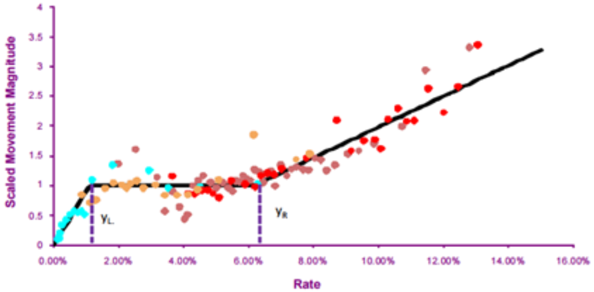


Figure 11: Blue: Japanese Yen, Red: Sterling, Maroon: US $, Orange: Swiss Francs

Source: Deguillaume, Rebonato, Pogudin

Deguillaume, Rebonato, Pogudin: *The Nature of the Dependence of Magnitude of Rate Moves on the Rates Levels: A Universal Relationship*

Calibrating interest rate models without analytical formulae

Gradient descent on Monte Carlo simulations

Traditional way to calibrate models: Derive analytic (approximate) swaption prices

Analytic prices turn out to be unnecessary when we apply a trick or two from deep learning‵s large scale optimizations

Procedure:

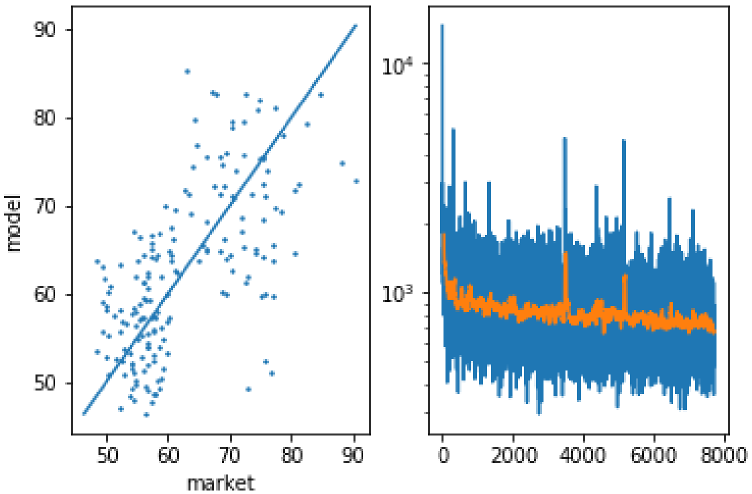
- Use market data (e.g. monthly 2003-2017).
- Using ~10 random valuation dates and ~100 paths each, run a Monte Carlo simulation with appropriate time discretisation (e.g. 6-24 steps/year and predictor/corrector method)
- Calculate Monte Carlo swaption prices and volatilities and error
- Use automatic differentiation framework to differentiate error by parameters
- Use stochastic gradient descent (-type) optimization

Also works with a single valuation date, then it is a lot less stochastic.

Calibrating interest rate models without analytical formulae

Some technical considerations

- With GPU computing, can compute gradient descent step in ~1 second or so on a commodity GPU.
- With single calibration date, a 50-500 steps are ample.
- Need to separate random numbers (which we do not know how to differentiate) from formulas involving parameters (which we would like to do gradient descent on)
  - Relatively natural with SDE formulation
  - Known in deep learning as the "reparametrisation trick" popularised by Kingma & Welling
- Gradient estimate may be biased, but likely is consistent.
  → Using 50-100 paths per valuation date seems sufficient, but 1-10 probably not.



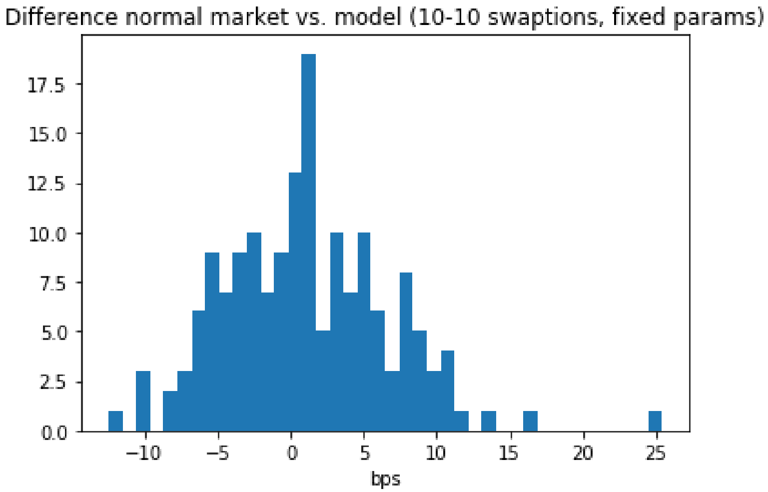
D. Kingma, M. Welling: Auto-Encoding Variational Bayes

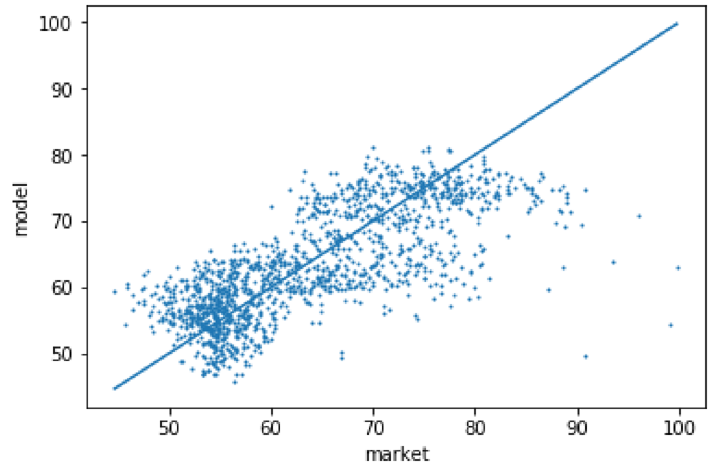
When offering many parameters

Calibration to 2003-2017 EUR Swap and 5,10,15x5,10,15 Swaptions

Procedure:

- Use candidate model.
- Fix lower bound
- Add lots of variable parameters to level dependency of volatility.
- Perform gradient descent.
- Evaluate difference market vs. model normal volatility for a 10-10 at the money swaption for each date.





Results:

- Much better fit to swaptions than baseline more than 50% within 5bps and more than 80% within 10bps across 5,10,15x5,10,15 swaptions, even better for 10x10
- However…

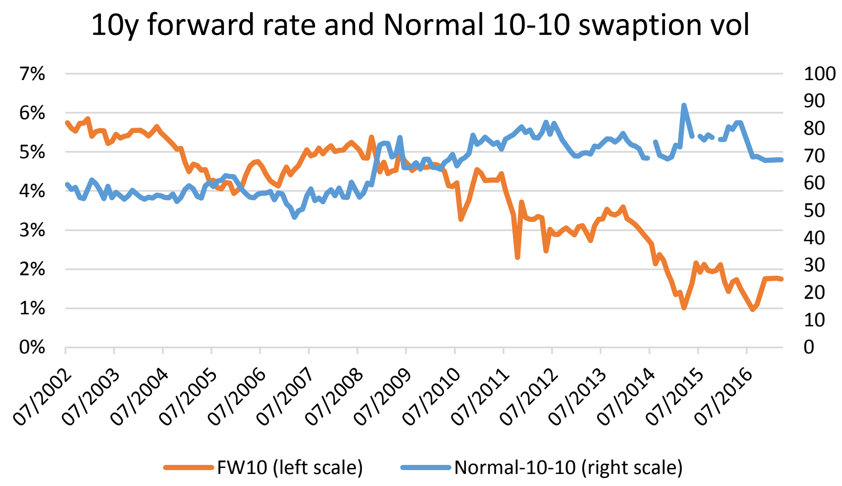
The need for date-specific volatility parameters

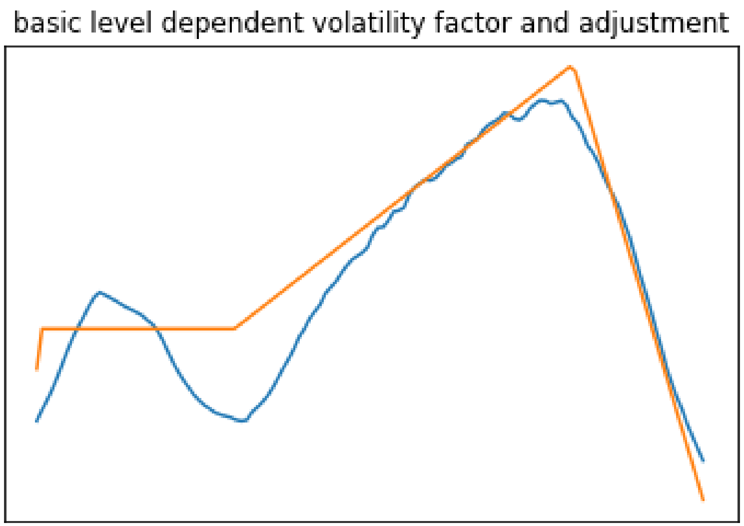
Need stochastic / calibration date specific volatility parameters

- Evidence in the literature (also in the two references above)
- in our data: looking at <=2008 and >=2009 will lead to non-monotone dependence without factors beyond level-dependence (see right graphic)

However we should likely use it

- only after/while considering structural dependencies (similar to Filipović et al.)
- probably not by following typical calibration practice of reproducing a single day’s smile (e.g. in Brigo / Mercurio‘s book, but also in the wild)





Conclusions

Having a view on parameters and state is highly recommended:

- It makes calibration more robust.
- Simplifies as-if calculations such as interest rate sensitivities in Solvency II, planning scenarios
- Produces less unexplained changes when comparing valuation dates.

Gradient descent through Monte-Carlo pricing works.

- Analytical pricing formulas probably not as important as they used to be.
- We don’t need to rely on approximation formulas for calibration and worry about their inaccuracy.
- Not needing analytical pricing formulas gives us more liberty to construct models with desirable properties.

Your questions and comments

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All opinions and assessments in this talk are my own, and do not constitute Deloitte advice.